ECON 210: Section 8

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Plan for today

- Non-stationary consumption-savings with normalization (Gourinchas and Parker (2002))

Consumption-Savings with Non-Stationarity

Gourinchas and Parker (2002)

We are interested in solving the problem

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{T}} E\left[\sum_{t=0}^{T} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}\right]$$

$$\text{s.t. } c_t + a_{t+1} \leq Ra_t + y_t$$

$$a_{t+1} \geq 0$$

$$a_{-1} \text{ given}$$

$$y_t = A_t P_t$$

$$\ln(P_t) = \ln(P_{t-1}) + \epsilon_t$$

- A_t is a deterministic age profile determinant of income.
- P_t are permanent shocks to income.
- Cash-on-hand w_t is defined as $w_t := Ra_t + y_t = Ra_t + A_t P_t$.

Consumption-Savings with Non-Stationarity

Gourinchas and Parker (2002)

The Bellman equation for this problem is

$$V_t\left(w_t, P_t\right) = \max_{c_t} \frac{c_t^{1-\gamma}}{1-\gamma} + \beta E\left[V_{t+1}\left(w_{t+1}, P_{t+1}\right) \mid P_t\right]$$

$$\text{s.t. } w_{t+1} = R\left(w_t - c_t\right) + A_{t+1} P_{t+1}$$

$$w_t - c_t \geq 0$$

- This problem has 3 state variables (which?)
- We will show we can solve an equivalent problem with 2 state variables.

Consumption-Savings with Non-Stationarity

Gourinchas and Parker (2002)

- How to do it? Problem scales with $P_t^{1-\gamma}$.
- Guess that $V_t(w_t, P_t)$ is homogenous of degree $1-\gamma$ in P_t, i.e.,

$$V_t(w_t, P_t) = V_t\left(\frac{w_t}{P_t} P_t, P_t\right) = \underbrace{V_t\left(\frac{w_t}{P_t}, 1\right)}_{:= \hat{V}_t(\hat{w}_t)} P_t^{1-\gamma} \tag{1}$$

- Plug this into the Bellman equation and check that we still get a recursive representation.

Consumption-Savings with Non-Stationarity

Gourinchas and Parker (2002)

Confirming our conjecture, we obtain the following Bellman equation

$$\hat{V}_t\left(\hat{w}_t\right) = \max_{\hat{c}_t} \frac{\hat{c}_t^{1-\gamma}}{1-\gamma} + \beta E_{\epsilon_{t+1}}\left[\hat{V}_{t+1}\left(\hat{w}_{t+1}\right)\left(\exp(\epsilon_{t+1})\right)^{1-\gamma}\right]$$

$$\text{s.t. } \hat{w}_{t+1} = \frac{R\left(\hat{w}_t - \hat{c}_t\right)}{e^{\epsilon_{t+1}}} + A_{t+1}$$

$$\hat{w}_t - \hat{c}_t \geq 0$$

- Note we take expectations over ϵ_{t+1}.

Solution Algorithm

1. Discretize grid for ϵ_{t+1}. Use Tauchen method with persistence $= 0$ and obtain the transition matrix Π.
2. Discretize grid for $\hat{w}_t$ and $\hat{c}_t$.
3. Set terminal value function to the optimal one, i.e., $\hat{V}_T(\hat{w}_T) = \frac{\hat{w}^{1-\gamma}}{1-\gamma}$.
4. Compute optimal policy and value function for period $T-1$.
   i) Fix $\hat{w}_i$
   ii) For each possible consumption choice $\hat{c}_j$ in the grid

$$T_{ij} = \frac{\hat{c}_j^{1-\gamma}}{1-\gamma} + \beta \sum_{m'} \hat{V}_T \underbrace{\left(\frac{R\left(\hat{w}_i - \hat{c}_j\right)}{e^{m'}} + A_T \right)}_{\hat{w}_T} \left(e^{m'}\right)^{1-\gamma} \Pi\left(m'\right)$$

   iii) Set $\hat{V}_{T-1}(\hat{w}_i) = \max_j T_{ij}$.
      - Will have to search for grid point closest to $\hat{w}_T$.
      - Note that all rows of Π are the same.
5. Having computed this for every point in the state space, iterate backwards to $T-2$ and repeat until time 0.

Gourinchas, Pierre-Olivier and Jonathan A Parker, "Consumption over the life cycle," *Econometrica*, 2002, *70* (1), 47–89.